Physics 04-03 Elastic and Inelastic Collisions   Name: ______________________

Kinetic Energy

- Energy of **motion**
- $KE = \frac{1}{2}mv^2$

Subatomic – **kinetic** energy often **conserved**

Macroscopic – **kinetic** energy usually not **conserved**

- Converted into **heat**
- Converted into **distortion** or **damage**

Elastic – **kinetic** energy **conserved**

Inelastic – **kinetic** energy **Not conserved**

Completely inelastic – the objects **stick** together

You are playing marbles. Your 0.10 kg shooter traveling at 1 m/s hits a stationary 0.05 kg cat's eye marble. If it is an elastic collision what are the velocities after the collision?

Momentum

$$m_s v_{0s} + m_c v_{0c} = m_s v_{fs} + m_c v_{fc}$$

$$(0.1\ kg)\left(1\frac{m}{s}\right) + (0.05\ kg)(0) = (0.1\ kg)v_{fs} + (0.05\ kg)v_{fc}$$

$$0.1\ kg\frac{m}{s} = (0.1\ kg)v_{fs} + (0.05\ kg)v_{fc}$$

$$v_{fs} = 1\ m/s - 0.5\ v_{fc}$$

Kinetic Energy

$$\frac{1}{2}m_s v_{0s}^2 + \frac{1}{2}m_c v_{0c}^2 = \frac{1}{2}m_s v_{fs}^2 + \frac{1}{2}m_c v_{fc}^2$$

$$\frac{1}{2}(0.1\ kg)\left(1\frac{m}{s}\right)^2 + 0 = \frac{1}{2}(0.1\ kg)v_{fs}^2 + \frac{1}{2}(0.05\ kg)v_{fc}^2$$

$$0.05\ J = (0.05\ kg)v_{fs}^2 + (0.025\ kg)v_{fc}^2$$

$$v_{fs}^2 + 0.5\ v_{fc}^2 = 1\ \left(\frac{m}{s}\right)^2$$

Substitution

$$\left(1\frac{m}{s} - 0.5v_{fc}\right)^2 + 0.5\ v_{fc}^2 = 1\ \left(\frac{m}{s}\right)^2$$

$$1\ \left(\frac{m}{s}\right)^2 - \left(1\frac{m}{s}\right)v_{fc} + 0.25\ v_{fc}^2 + 0.5\ v_{fc}^2 = 1\ \left(\frac{m}{s}\right)^2$$

$$-\left(1\frac{m}{s}\right)v_{fc} + 0.75\ v_{fc}^2 = 0$$

$$v_{fc}\left(-1\frac{m}{s} + 0.75\ v_{fc}\right) = 0$$

$$v_{fc} = 0\ or\ 1.33\ m/s$$

$$v_{fs} = 1\frac{m}{s} - 0.5\ v_{fc}$$

$$V_{fs} = 1\ m/s - 0.5(1.33\ m/s) = 0.333\ m/s$$

Police will sometime reconstruct car accidents. In one accident, the cars stuck together and slid 12 m before they stopped. They measure the coefficient of friction as 0.70. The blue car's mass is 1100 kg and was sitting still at a stop sign when it was hit by the red car whose mass is 990 kg. How fast was the red car going when it hit the blue car?

Work backwards

Find acceleration after collision:

$$v^2 = v_0^2 + 2a(d - d_0)$$

$$0^2 = v_0^2 + 2a(12\ m - 0m)$$

$$-\frac{v_0^2}{24\ m} = a$$

Forces (cars stuck together m=2090 kg):

$$\mu F_N = ma$$

$$-(0.70)(mg) = ma$$

$$-(0.70)\left(9.8\frac{m}{s^2}\right) = -\frac{v_0^2}{24\ m}$$

$$164.64\frac{m^2}{s^2} = v_0^2$$

$$v_0 = 12.83\frac{m}{s}$$

Use momentum (v_0 is now final v):

$$m_1 v_{01} + m_2 v_{02} = m_1 v_1 + m_2 v_2$$

$$(1100\ kg)\left(0\frac{m}{s}\right) + (990\ kg)v_{02} = (1100\ kg)\left(12.83\frac{m}{s}\right) + (990\ kg)\left(12.83\frac{m}{s}\right)$$

$$(990\ kg)v_{02} = 26817.2\ kg\frac{m}{s}$$

$$v_{02} = 27.1\frac{m}{s}$$

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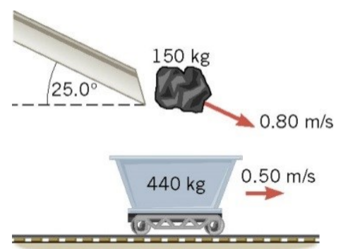
Practice Work

1. In an elastic collision, is the kinetic energy of *each* object the same before and after the collision? Explain.
2. What is an elastic collision?
3. What is an inelastic collision? What is a perfectly inelastic collision?
4. Mixed-pair ice skaters performing in a show are standing motionless at arms length just before starting a routine. They reach out, clasp hands, and pull themselves together by only using their arms. Assuming there is no friction between the blades of their skates and the ice, what is their velocity after their bodies meet?
5. In a football game, a receiver is standing still, having just caught a pass. Before he can move, a tackler, running at a velocity of +4.5 m/s, grabs him. The tackler holds onto the receiver, and the two move off together with a velocity of +2.6 m/s. The mass of the tackler is 115 kg. Assuming that momentum is conserved, find the mass of the receiver. (Cutnell 7.25) **84 kg**
6. A 1055-kg van, stopped at a traffic light, is hit directly in the rear by a 715-kg car traveling with a velocity of +2.25 m/s. Assume that the transmission of the van is in neutral, the brakes are not being applied, and the collision is elastic. What is the final velocity of (a) the car and (b) the van? (Cutnell 7.26) **−0.432 m/s, 1.82 m/s**
7. A cue ball (mass = 0.165 kg) is at rest on a frictionless pool table. The ball is hit dead center by a pool stick, which applies an impulse of +1.50 Ns to the ball. The ball then slides along the table and makes an elastic head-on collision with a second ball of equal mass that is initially at rest. Find the velocity of the second ball just after it is struck. (Cutnell 7.29) **9.09 m/s**
8. A 5.00-kg ball, moving to the right at a velocity of +2.00 m/s on a frictionless table, collides head-on with a stationary 7.50-kg ball. Find the final velocities of the balls if the collision is (a) elastic and (b) completely inelastic. (Cutnell 7.31) **−0.4 m/s, 1.6 m/s; 0.8 m/s**
9. A mine car, whose mass is 440 kg, rolls at a speed of 0.50 m/s on a horizontal track, as the drawing shows. A 150-kg chunk of coal has a speed of 0.80 m/s when it leaves the chute. Determine the velocity of the car/coal system after the coal has come to rest in the car. (Cutnell 7.34) **0.56 m/s**



10. A 30,000-kg freight car is coasting at 0.850 m/s with negligible friction under a hopper that dumps 110,000 kg of scrap metal into it. (a) What is the final velocity of the loaded freight car? (b) How much kinetic energy is lost? (OpenStax 8.36) **0.182 m/s,** $\mathbf{8.52 \times 10^3}$ **J**
11. During an ice show, a 60.0-kg skater leaps into the air and is caught by an initially stationary 75.0-kg skater. (a) What is their final velocity assuming negligible friction and that the 60.0-kg skater's original horizontal velocity is 4.00 m/s? (b) How much kinetic energy is lost? (OpenStax 8.32) **1.78 m/s, −267 J**
12. An automobile has a mass of 2100 kg and a velocity of +17 m/s. It makes a rear-end collision with a stationary car whose mass is 1900 kg. The cars lock bumpers and skid off together with the wheels locked. (a) What is the velocity of the two cars just after the collision? (b) Find the impulse (magnitude and direction) that acts on the skidding cars from just after the collision until they come to a halt. (c) Review: If the coefficient of kinetic friction between the wheels of the cars and the pavement is $\mu_k = 0.68$, determine how far the cars skid before coming to rest. (Cutnell 7.33) **8.9 m/s,** $\mathbf{-3.6 \times 10^4}$ **Ns, 6.0 m**

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Worked-Out Solutions	

1. No, the total KE of all the objects is the same before and after the collision.
2. An elastic collision means the total KE of all the objects is the same before and after the collision.
3. An inelastic collision means the total KE of all the objects is not the same before and after the collision. A perfectly inelastic collision means the total KE of all the objects is not the same before and after the collision.
4. 0, The total momentum before was 0, so the total momentum afterwards is 0.
5. $m_1 v_{10} + m_2 v_{20} = m_1 v_{1f} + m_2 v_{2f} \rightarrow m_1 \left(0 \frac{m}{s}\right) + (115\ kg)\left(4.5 \frac{m}{s}\right) = m_1 \left(2.6 \frac{m}{s}\right) + (115\ kg)\left(2.6 \frac{m}{s}\right) \rightarrow$
   $517.5\ kg \frac{m}{s} = m_1 \left(2.6 \frac{m}{s}\right) + 299\ kg \frac{m}{s} \rightarrow 218.5\ kg \frac{m}{s} = m_1 \left(2.6 \frac{m}{s}\right) \rightarrow m_1 = \mathbf{84\ kg}$
6. Momentum: $m_1 v_{10} + m_2 v_{20} = m_1 v_{1f} + m_2 v_{2f} \rightarrow (1055\ kg)\left(0 \frac{m}{s}\right) + (715\ kg)\left(2.25 \frac{m}{s}\right) = (1055\ kg) v_{vf} + (715\ kg) v_{cf} \rightarrow$
   $2.25 \frac{m}{s} = 1.4755 v_{vf} + v_{cf} \rightarrow v_{cf} = 2.25 \frac{m}{s} - 1.4755 v_{vf}$
   KE: $\frac{1}{2} m_1 v_{10}^2 + \frac{1}{2} m_2 v_{20}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2 \rightarrow m_1 v_{10}^2 + m_2 v_{20}^2 = m_1 v_{1f}^2 + m_2 v_{2f}^2 \rightarrow$
   $(1055\ kg)\left(0 \frac{m}{s}\right)^2 + (715\ kg)\left(2.25 \frac{m}{s}\right)^2 = (1055\ kg) v_{vf}^2 + (715\ kg) v_{cf}^2 \rightarrow 5.0625 \frac{m^2}{s^2} = 1.4755 v_{vf}^2 + v_{cf}^2$
   Substitute: $5.0625 \frac{m^2}{s^2} = 1.4755 v_{vf}^2 + \left(2.25 \frac{m}{s} - 1.4755 v_{vf}\right)^2 \rightarrow$
   $5.0625 \frac{m^2}{s^2} = 1.4755\ v_{vf}^2 + 5.0625 \frac{m^2}{s^2} - \left(6.63975 \frac{m}{s}\right) v_{vf} + 2.1771 v_{vf}^2 \rightarrow 0 = 3.6526 v_{vf}^2 - \left(6.63975 \frac{m}{s}\right) v_{vf} \rightarrow$
   $0 = v_{vf}\left(3.6526 v_{vf} - 6.63975 \frac{m}{s}\right) \rightarrow 0 = 3.6526 v_{vf} - 6.63975 \frac{m}{s} \rightarrow v_{vf} = \mathbf{1.82 \frac{m}{s}}$
   $v_{cf} = 2.25 \frac{m}{s} - 1.4755\left(1.82 \frac{m}{s}\right) \rightarrow v_{cf} = \mathbf{-0.432 \frac{m}{s}}$
7. First ball: $J = m v_f - m v_0 \rightarrow 1.50\ Ns = (0.165\ kg) v_f - (0.165\ kg)\left(0 \frac{m}{s}\right) \rightarrow v_f = 9.09 \frac{m}{s}$
   Collision: Momentum: $m_1 v_{10} + m_2 v_{20} = m_1 v_{1f} + m_2 v_{2f} \rightarrow$
   $(0.165\ kg)\left(9.09 \frac{m}{s}\right) + (0.165\ kg)\left(0 \frac{m}{s}\right) = (0.165\ kg) v_{1f} + (0.165\ kg) v_{2f} \rightarrow 9.09 \frac{m}{s} = v_{1f} + v_{2f} \rightarrow v_{1f} = 9.09 \frac{m}{s} - v_{2f}$
   Kinetic Energy: $\frac{1}{2} m_1 v_{10}^2 + \frac{1}{2} m_2 v_{20}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2 \rightarrow$
   $\frac{1}{2}(0.165\ kg)\left(9.09 \frac{m}{s}\right)^2 + \frac{1}{2}(0.165\ kg)\left(0 \frac{m}{s}\right)^2 = \frac{1}{2}(0.165\ kg) v_{1f}^2 + \frac{1}{2}(0.165\ kg) v_{2f}^2 \rightarrow 82.64 \frac{m^2}{s^2} = v_{1f}^2 + v_{2f}^2 \rightarrow$
   Substitution: $82.64 \frac{m^2}{s^2} = \left(9.09 \frac{m}{s} - v_{2f}\right)^2 + v_{2f}^2 \rightarrow 82.64 \frac{m^2}{s^2} = 82.64 \frac{m^2}{s^2} - \left(18.18 \frac{m}{s}\right) v_{2f} + v_{2f}^2 + v_{2f}^2 \rightarrow$
   $0 = 2 v_{2f}^2 - \left(18.18 \frac{m}{s}\right) v_{2f} \rightarrow 0 = 2 v_{2f}^2 \left(v_{2f} - 9.09 \frac{m}{s}\right) \rightarrow v_{2f} - 9.09 \frac{m}{s} = 0 \rightarrow v_{2f} = \mathbf{9.09 \frac{m}{s}}$
8. (a) Momentum: $m_1 v_{10} + m_2 v_{20} = m_1 v_{1f} + m_2 v_{2f} \rightarrow$
   $(5.00\ kg)\left(2.00 \frac{m}{s}\right) + (7.50\ kg)\left(0 \frac{m}{s}\right) = (5.00\ kg) v_{1f} + (7.50\ kg) v_{2f} \rightarrow 2.00 \frac{m}{s} = v_{1f} + 1.5 v_{2f} \rightarrow v_{1f} = 2.00 \frac{m}{s} - 1.5 v_{2f}$
   Kinetic Energy: $\frac{1}{2} m_1 v_{10}^2 + \frac{1}{2} m_2 v_{20}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2 \rightarrow$
   $\frac{1}{2}(5.00\ kg)\left(2.00 \frac{m}{s}\right)^2 + \frac{1}{2}(7.50\ kg)\left(0 \frac{m}{s}\right)^2 = \frac{1}{2}(5.00\ kg) v_{1f}^2 + \frac{1}{2}(7.50\ kg) v_{2f}^2 \rightarrow 4 \frac{m^2}{s^2} = v_{1f}^2 + 1.5 v_{2f}^2$
   Substitution: $4 \frac{m^2}{s^2} = \left(2 \frac{m}{s} - 1.5\ v_{2f}\right)^2 + 1.5 v_{2f}^2 \rightarrow 4 \frac{m^2}{s^2} = 4 \frac{m^2}{s^2} - 6 \frac{m}{s} v_{2f} + 2.25 v_{2f}^2 + 1.5 v_{2f}^2 \rightarrow 0 = 3.75 v_{2f}^2 - 6 \frac{m}{s} v_{2f} \rightarrow$
   $0 = v_{2f}\left(3.75\ v_{2f} - 6 \frac{m}{s}\right) \rightarrow 3.75 v_{2f} - 6 \frac{m}{s} = 0 \rightarrow v_{2f} = \mathbf{1.6 \frac{m}{s}}$
   $v_{1f} = 2.00 \frac{m}{s} - 1.5\left(1.6 \frac{m}{s}\right) \rightarrow v_{1f} = \mathbf{-0.4 \frac{m}{s}}$
   (b) Momentum: $m_1 v_{10} + m_2 v_{20} = m_1 v_{1f} + m_2 v_{2f} \rightarrow$
   $(5.00\ kg)\left(2.00 \frac{m}{s}\right) + (7.50\ kg)\left(0 \frac{m}{s}\right) = (5.00\ kg) v_f + (7.50\ kg) v_f \rightarrow 10\ kg \frac{m}{s} = (12.50\ kg) v_f \rightarrow v_f = \mathbf{0.8 \frac{m}{s}}$
9. $m_1 v_{10} + m_2 v_{20} = m_1 v_{1f} + m_2 v_{2f} \rightarrow (440\ kg)\left(0.50 \frac{m}{s}\right) + (150\ kg)\left(0.80 \frac{m}{s} \cos 25°\right) = (440\ kg) v_f + (150\ kg) v_f \rightarrow$
   $328.76\ kg \frac{m}{s} = (590\ kg) v_f \rightarrow v_f = \mathbf{0.56 \frac{m}{s}}$

10. (a) $m_1v_{10} + m_2v_{20} = m_1v_{1f} + m_2v_{2f} \rightarrow$
$(30000\ kg)\left(0.850\frac{m}{s}\right) + (110000\ kg)\left(0\frac{m}{s}\right) = (30000\ kg)v_f + (110000\ kg)v_f \rightarrow 25500\ kg\frac{m}{s} = (140000\ kg)v_f \rightarrow$
$v_f = \mathbf{0.182}\frac{\boldsymbol{m}}{\boldsymbol{s}}$
(b) $\Delta KE = \left(\frac{1}{2}m_1v_{1f}^2 + \frac{1}{2}m_2v_{2f}^2\right) - \left(\frac{1}{2}m_1v_{10} + \frac{1}{2}m_2v_{20}\right) \rightarrow$
$\Delta KE = \left(\frac{1}{2}(30000\ kg)\left(0.182\frac{m}{s}\right)^2 + \frac{1}{2}(110000\ kg)\left(0.182\frac{m}{s}\right)^2\right) - \left(\frac{1}{2}(30000\ kg)\left(0.850\frac{m}{s}\right)^2 + \frac{1}{2}(110000\ kg)\left(0\frac{m}{s}\right)^2\right)$
$\Delta KE = \mathbf{-8.52 \times 10^3}\ \boldsymbol{J}$

11. (a) $m_1v_{10} + m_2v_{20} = m_1v_{1f} + m_2v_{2f} \rightarrow$
$(60\ kg)\left(4\frac{m}{s}\right) + (75\ kg)\left(0\frac{m}{s}\right) = (60\ kg)v_f + (75\ kg)v_f \rightarrow 240\ kg\frac{m}{s} = (135\ kg)v_f \rightarrow v_f = \mathbf{1.78}\frac{\boldsymbol{m}}{\boldsymbol{s}}$
(b) $\Delta KE = \left(\frac{1}{2}m_1v_{1f}^2 + \frac{1}{2}m_2v_{2f}^2\right) - \left(\frac{1}{2}m_1v_{10} + \frac{1}{2}m_2v_{20}\right) \rightarrow$
$\Delta KE = \left(\frac{1}{2}(60\ kg)\left(1.78\frac{m}{s}\right)^2 + \frac{1}{2}(75\ kg)\left(1.78\frac{m}{s}\right)^2\right) - \left(\frac{1}{2}(60\ kg)\left(4\frac{m}{s}\right)^2 + \frac{1}{2}(75\ kg)\left(0\frac{m}{s}\right)^2\right)$
$\Delta KE = \mathbf{-267}\ \boldsymbol{J}$

12. (a) $m_1v_{10} + m_2v_{20} = m_1v_{1f} + m_2v_{2f} \rightarrow$
$(2100\ kg)\left(17\frac{m}{s}\right) + (1900\ kg)\left(0\frac{m}{s}\right) = (2100\ kg)v_f + (1900\ kg)v_f \rightarrow 35700\ kg\frac{m}{s} = (4000\ kg)v_f \rightarrow v_f = \mathbf{8.9}\frac{\boldsymbol{m}}{\boldsymbol{s}}$
(b) $J = \Delta p \rightarrow J = (m_1v_{1f} + m_2v_{2f}) - (m_1v_{10} + m_2v_{20}) \rightarrow$
$J = \left((2100\ kg)\left(0\frac{m}{s}\right) + (1900\ kg)\left(0\frac{m}{s}\right)\right) - \left((2100\ kg)\left(8.9\frac{m}{s}\right) + (1900\ kg)\left(8.9\frac{m}{s}\right)\right) \rightarrow J = \mathbf{-3.6 \times 10^4}\ \boldsymbol{Ns}$
(c) y:$F_{net} = ma \rightarrow N - W = 0 \rightarrow N = W = mg$
x: $F_{net} = ma \rightarrow -\mu N = ma \rightarrow -\mu mg = ma \rightarrow -\mu g = a \rightarrow -(0.68)\left(9.8\frac{m}{s^2}\right) = a \rightarrow a = -6.664\frac{m}{s^2}$
$v_f^2 = v_0^2 + 2a(x - x_0) \rightarrow \left(0\frac{m}{s}\right)^2 = \left(8.9\frac{m}{s}\right)^2 + 2\left(-6.664\frac{m}{s^2}\right)(x) \rightarrow -79.66\frac{m^2}{s^2} = \left(-13.328\frac{m}{s^2}\right)x \rightarrow x = \mathbf{6.0}\ \boldsymbol{m}$